Matrices and Determinants

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Matrix Definition

- A *matrix* is a rectangular array of numbers

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

•*Elements*

•Row

•Column

•*Order*

•*square matrix*

•*main diagonal*

Matrix Definition

- The sum of the diagonal elements of a square matrix is called the *trace* of A*
- A matrix in which every element is zero, is called a *zero matrix*.

Matrix Operations

- If A and B are two matrices conformable for addition (subtraction), their sum (difference) will be

$$C = A \pm B = [a_{ij} \pm b_{ij}]$$

with the same order

Ex.1

- Compute A+B and A-B given that

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 3 & 0 \\ -1 & 2 & 5 \end{bmatrix}$$

Matrix Operations

- If ***k*** is any scalar, then multiplication of a matrix ***A*** by the scalar ***k***, is the multiplication of every element of ***A*** by ***k***.

Ex.2

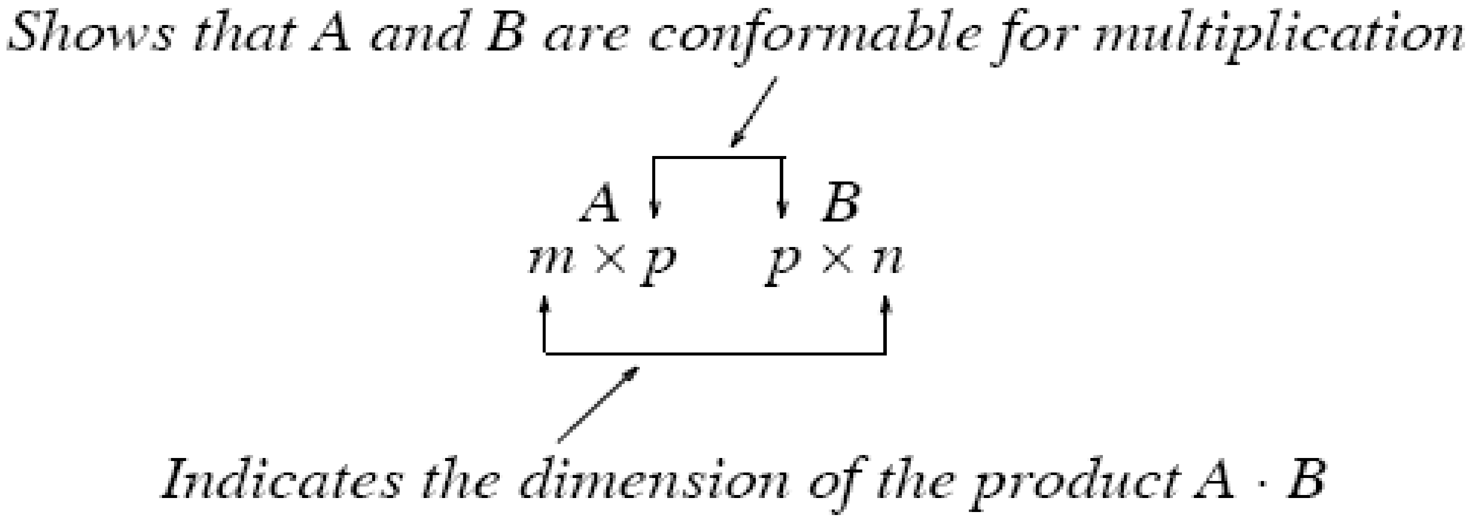
- Multiply the matrix

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$$

by ***k***=5

Matrix Operations

- Two matrices ***A*** and ***B*** are *conformable for multiplication* ***A·B*** in that order, only when the number of columns of matrix ***A*** is equal to the number of rows of matrix ***B.***



Ex.3

- Given that

$$C = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

compute the products ***C·D*** and ***D·C***

Upper Triangular Matrix

- A square matrix is said to be *upper triangular* when all the elements below the diagonal are zero.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a_{22} & a_{23} & \dots & a_{2n} \\ 0 & 0 & \dots & \dots & \dots \\ \dots & \dots & 0 & \dots & \dots \\ 0 & 0 & 0 & \dots & a_{mn} \end{bmatrix}$$

Lower Triangular Matrix

- A square matrix is said to be *lower triangular* when all the elements above the diagonal are zero.

$$B = \begin{bmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Ex.4

- Write a program to display an $n \times n$ *upper triangular* matrix with random number.

Diagonal Matrix

- A square matrix is said to be *diagonal maxtrix*, if all elements are zero, except those in the diagonal.

$$C = \begin{bmatrix} a_{11} & 0 & 0 & \dots & 0 \\ 0 & a_{22} & 0 & \dots & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \dots & a_{mn} \end{bmatrix}$$

Scaler Matrix

- A diagonal matrix is called a *scalar matrix*, *if all elements in diagonal are equal to k, k is scalar, ex. as k=4,*

$$D = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Identity Matrix

- A scalar matrix with *k=1*, is called an *identity matrix* ***I***.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ex.5

- Write a program to display an $n \times n$ identity matrix.